ERRATUM FOR "DELIGNE'S NOTES ON NAGATA COMPACTIFICATIONS"

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There are two small (but fixable) errors in [C] (entirely my fault) that we address here. I am grateful to Andrea Parri and Johan de Jong for pointing out these respective mistakes that I made.

1. The first error occurs in the final few paragraphs of Case 1 of the proof of [C, Thm. 2.4]. I made an incorrect choice of the ideal along which to form a certain blow-up. The problem is that the map as I wrote it on the blow-up along this ideal does not make sense. To give a correct choice of ideal for which the required map makes sense with the properties that are needed, we proceed as follows. In the notation used there, the ideal $I \subseteq A$ should have been defined to be the ideal $I = (u_i^N u_{i'}^N) + (g_{i,i',j})$ generated by all products $u_i^N u_{i'}^N$ and all $g_{i,i',j}$ (and $\mathscr{I}$ is taken to be the quasi-coherent ideal sheaf on $X = \mathrm{Spec}(A)$ associated to this new choice of ideal), rather than the ideal $(u_i^N) + (g_{i,i',j})$ generated by all u_i^N and $g_{i,i',j}$ as in [C].

[The problem is that the construction of the X-map $\widetilde{f} : \widetilde{X} = \mathrm{Bl}_{\mathscr{I}}(X) \to \mathbf{P}^n_X$ extending the given X-map $f : U \to \mathbf{A}^n_X$ does not make sense using the original choice of I because on the chart $\mathrm{Spec}(A[I(u_i^N)^{-1}])$ under the original definition an A-map was "defined" to go from this chart to $\mathrm{Spec}\, A[T_0/T_n, \dots, T_{n-1}/T_n]$ by $T_j/T_n \mapsto g_{i,i,j}/u_i^{2N}$, which does not make sense. More precisely, the fraction $g_{i,i,j}/u_i^{2N}$ only makes sense in $A[I(u_i^N)^{-1}]$ when it is expressed in "reduced form" as g_{ij}/u_i^N (recalling that $g_{i,i,j}$ is defined to be $u_i^N g_{ij}$), but with this change in the fraction the calculation to check agreement on overlaps for the local definition of $\widetilde{f}$ on standard open charts of the blow-up does not work. That calculation was omitted as an exercise for the reader.]

Using the corrected choice of ideal I as indicated above, the definition of the X-map $\widetilde{f} : \widetilde{X} \to \mathbf{P}^n_X$ can be modified to make sense on this new blow-up $\widetilde{X}$ as follows. We work with the open covering of $\widetilde{X}$ given by the standard blow-up charts $\mathrm{Spec}(A[I(u_i^N u_{i'}^N)^{-1}])$ and $\mathrm{Spec}(A[I g_{i,i',j}^{-1}])$, and define the A-map $\mathrm{Spec}(A[I g_{i',i',j}^{-1}]) \to D_+(T_j)$ exactly as in the original proof and the A-map $\mathrm{Spec}(A[I(u_i^N u_{i'}^N)^{-1}]) \to \mathbf{A}^n_X \simeq D_+(T_n)$ by $T_j/T_n \mapsto g_{i,i',j}/(u_i^N u_{i'}^N)$ for all $0 \le j \le n-1$. (In effect, the original version of the proof in [C] used this latter chart only for $i' = i$.)

The above ideal and explicit formulas defining $\widetilde{f}$ are never used outside of the context of Case 1 of the proof of [C, Thm. 2.4], so this correction does not affect anything else in the paper.

2. The second error occurs at the end of the appendix (which is never used in the main text of the paper), in the proof of [C, Cor. A2]. I overlooked that X' (as opposed to X) might not be affine and I forgot to define the notation X_i. It was intended to arrange for X' to be affine, in which case we could rename X' as X and X'_i as X_i and the argument would then work as written.

Thus, we need to prove that for a closed immersion $X \hookrightarrow X'$ of schemes with X affine and X' quasi-compact and quasi-separated, there is a finitely presented closed subscheme $Z \subseteq X'$ containing X such that Z is affine. By absolute noetherian approximation [TT, Thm. C.9], $X' = \varprojlim X'_\alpha$ for an inverse system $\{X'_\alpha\}$ of finite type $\mathbf{Z}$-schemes with affine transition maps. Let $Z_\alpha \subseteq X'_\alpha$ be the schematic image of the affine map $X \to X'_\alpha$. Then $\{Z_\alpha\}$ is an inverse system of $\mathbf{Z}$-schemes of finite type with affine transition maps, and $\varprojlim Z_\alpha = X$. (Note that we do not claim that the natural map $Z_\beta \to Z_\alpha \times_{X'_\alpha} X'_\beta$ is an isomorphism for $\beta \ge \alpha$; this is generally false and we do not need it.) Since the Z_α are finite type over $\mathbf{Z}$ and their

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limit X is affine, it follows from [TT, Prop. C.6] that Z_{α_0} is affine for some large α_0. Hence, the pullback $Z = Z_{\alpha_0} \times_{X'_{\alpha_0}} X'$ is a closed subscheme of finite presentation in X' that contains X and is affine (as it is affine over Z_{α_0}).

References

[C] B. Conrad, "Deligne's notes on Nagata compactifications", *Journal of the Ramanujan Math. Soc.*, **22** (no. 3), 2007, pp. 205–257.

[TT] R. Thomason, T. Trobaugh, "Higher algebraic K-theory of schemes and of derived categories", in The Grothendieck Festschrift, Vol. III, pp. 247–435, Progress in Mathematics, Birkhäuser, Boston (1990).

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